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FURTHER TESTS OF A GRID SYSTEM FOR GLOBAL NUMERICAL PREDICTION

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ABSTRACT

Two smoothing techniques are tested as a practical means of allowing a larger time step in the numerical integration of a primitive equation free-surface model. The numerical integration uses a finite-difference grid and operators based on the method of Kurihara and Holloway.

A time step six times larger can be used with a corresponding six-fold decrease in computer time, by implementing the weighted averaging procedure given by Langlois and Kwok in their description of the Mintz-Arakawa general circulation model. A Fourier filtering scheme permits the use of a time step 10 times larger, and results in a five-fold improvement in computer time. After 10 days, the geopotential and wind fields obtained with these techniques still closely resemble the unsmoothed fields, the closest correspondence being found with the Fourier filtering technique.

In another set of experiments, steady-state solutions to special cases of the governing analytic equations are used as initial conditions in a test of the accuracy of the grid and operators. These steady-state solutions are preserved satisfactorily for the 10-day integration period.

1. INTRODUCTION

In a previous paper (Sankar-Rao and Umscheid 1969), we tested a numerical scheme based on the grid and finite-difference operators of Kurihara and Holloway (1967). The numerical model was of an incompressible homogeneous inviscid fluid in hydrostatic equilibrium, having a free surface. It was found that to achieve acceptable accuracy for the cases tested, the grid had to be modified by increasing the resolution near the poles. A similar result was obtained by Dey (1969) using real data.

The present investigation, conducted with the same numerical model, has two purposes. First, in our earlier study, the increased resolution required near the poles necessitated a proportional decrease in the time step used in the numerical integration. Here, we test two smoothing techniques as a practical means of allowing a larger time step without modifying the large scale features of the flow. Second, in another set of experiments, a test of the accuracy of the numerical scheme for extended predictions is made by using initial conditions which are steady-state solutions for special cases of the governing equations.

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2. MODEL EQUATIONS AND INITIAL CONDITIONS

The appropriate differential equations for the model atmosphere are

$$\frac{\partial \phi}{\partial t}=-\frac{\partial \phi u}{a \cos \theta\, \partial \lambda}-\frac{\partial \phi v \cos \theta}{a \cos \theta\, \partial \theta},$$

$$\frac{\partial \phi u}{\partial t}=-\frac{\partial \phi u^2}{a \cos \theta\, \partial \lambda}-\frac{\partial \phi u v \cos \theta}{a \cos \theta\, \partial \theta}+\left(f+\frac{\tan \theta u}{a}\right) v\phi-\phi \frac{\partial \phi}{a \cos \theta\, \partial \lambda}, \quad (1)$$

and

$$\frac{\partial \phi v}{\partial t}=-\frac{\partial \phi u v}{a \cos \theta\, \partial \lambda}-\frac{\partial \phi v^2 \cos \theta}{a \cos \theta\, \partial \theta}-\left(f+\frac{\tan \theta u}{a}\right) u\phi-\phi \frac{\partial \phi}{a\, \partial \theta},$$

where ϕ is geopotential, u and v are velocity components, f is twice the component of the rotating sphere's angular velocity about the local vertical as given below, θ is the latitude, λ is the longitude, and a is the radius of the globe. The finite-difference operators are identical to those used previously by Sankar-Rao and Umscheid (1969).

Two sets of initial conditions are utilized in the present experiments. For the smoothing tests (exp. 1, 2, and 3) the balanced initial conditions given by Phillips (1959*b*) are used. These were tested by Kurihara (1965), Grimmer and Shaw (1967), and Sankar-Rao and Umscheid (1969). Figure 1 depicts these initial conditions.

For experiments 4 and 5 the second set of initial conditions is used. They are steady-state analytic solutions of the equations having no rotation or with rotation of the globe about an axis through the two vortex centers. The initial conditions for experiment 4 are purely zonal flow and for experiment 5 are cross-polar flow.

The two flows are essentially the same but have different orientations to the finite-difference grid. The fields with purely zonal flow that serve as the initial conditions for experiment 4 are given by

$$u=u_0 \cos \theta,$$

$$v=0,$$

and

$$\phi=\phi_0-\left(a\Omega u_0+\frac{u_0^2}{2}\right) \sin^2 \theta.$$

In this case f in eq (1) is the Coriolis parameter, given by $f=2\Omega \sin \theta$. The initial conditions for experiment 5 are similar to those tested by Dey (1969) except that that study was for a nonrotating sphere. The fields with cross-polar flow used here are depicted in figure 2 and are given by

$$u=-u_0 \cos \lambda \sin \theta,$$

$$v=u_0 \sin \lambda,$$

and

$$\phi=\phi_0-\left(a\Omega u_0+\frac{u_0^2}{2}\right) \cos^2 \theta \cos^2 \lambda.$$

Here f in eq (1) is formally given by $f=2\Omega\cos\theta \cos\lambda$. The constants u_0 and ϕ_0 were taken as

$$u_0=5 \text{ m/s}$$

and

$$\phi_0=2.94\times10^4 \text{m}^2\text{s}^{-2}.$$

Also, Ω, the magnitude of the rotation as described above, has the value 7.292×10^{-5} s^{-1}.

3. SMOOTHING TECHNIQUES

Two smoothing techniques are tested in this study. The first technique applies weighted averaging of the form given by Langlois and Kwok (1969) in their description of the Mintz-Arakawa general circulation model. This averaging is over longitude and is accomplished by using

$$\bar{\psi}^m_{i,j}=\bar{\psi}^{m-1}_{i,j}+\tfrac{1}{2}\gamma_i(\bar{\psi}^{m-1}_{i,j-1}-2\bar{\psi}^{m-1}_{i,j}+\bar{\psi}^{m-1}_{i,j+1}) \qquad (2)$$

where ψ is the field to be smoothed, i is the index of latitude, and j is the index of longitude; $\bar{\psi}^m$ is the result of m averaging operations starting with the original

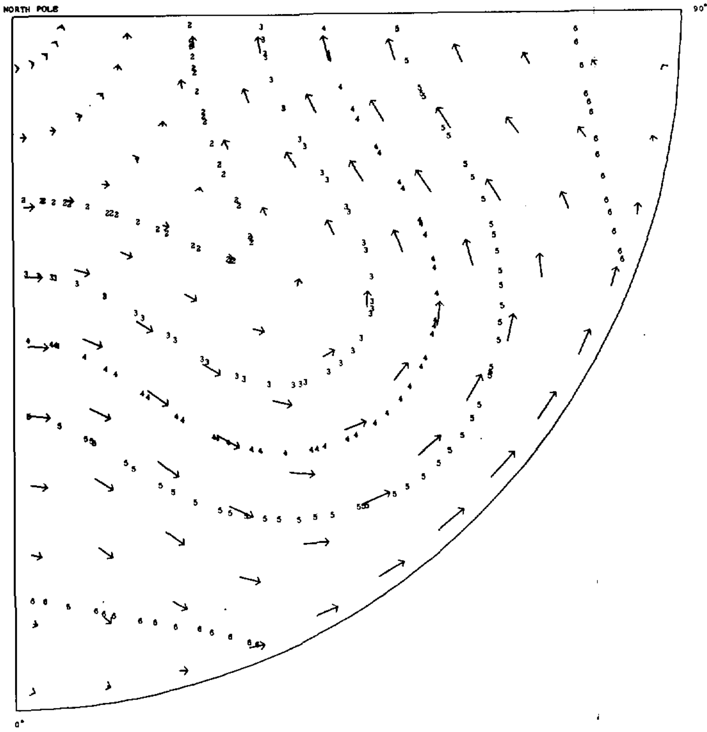

FIGURE 1.—Initial height field and wind vectors for one octant of the globe for experiments 1, 2, and 3. The computer-drawn height contours increase in value in steps of 0.5 km from contour 2=8.5 km.

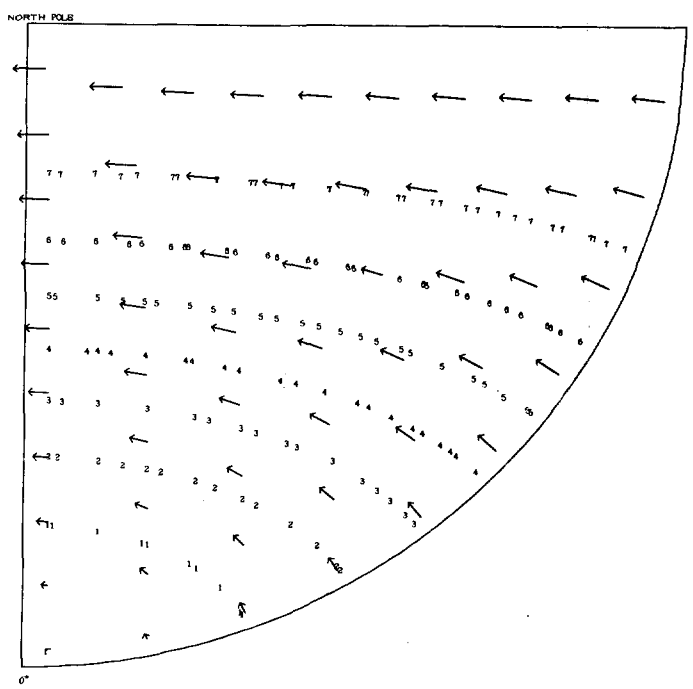

FIGURE 2.—Analytic steady-state solution used as initial conditions for experiment 5. The computer-drawn contours increase in value from contour 1=2.79 km in steps of 30 m.

TABLE 1.—*Smoothing parameters for modified Kurihara grid with resolution N=20, KP=8 (i.e. 8 points added at each lat.)*

θ	$\gamma/2$	D	M
(deg.)			
87.75	0.1189	11.46	11
83.25	0.1017	4.25	4
78.75	0.1137	2.82	2
74.25	0.07565	2.21	2

unsmoothed field $\bar{\psi}^0$. The weight γ_i is given by

$$\gamma_i=\frac{D_i-1}{4M_i}$$

where

$$D_i=\frac{\Delta\theta}{\cos\theta_i\Delta\lambda_i},$$

$M_i=\text{INT}(D_i)$ that is, the largest integer $\leq D_i$, and $\Delta\theta$ and $\Delta\lambda_i$ are grid intervals in latitude and longitude. In this study, smoothing is performed only for latitudes where $D_i\geq 2$. For these latitudes, table 1 gives the smoothing parameters as used in experiment 2. The objective of this form of averaging is to have the time step determined principally by the latitudinal rather than the minimum longitudinal grid spacing. To suppress waves moving in the longitudinal direction with wavelengths less than $\Delta\theta$, both the zonal flux (ϕu) in the divergence terms and the pressure gradient term in the u momentum equation are smoothed (Langlois and Kwok 1969). This can also be viewed as creating an effective $\Delta\lambda$ equal to $\Delta\theta$. In this study smoothing is carried out at every time step by applying eq (2) with $m=M_i$ where $D_i\geq 2$.

Neither the mean value of the field nor the wave number and phase of a Fourier component are changed by this type of smoothing. However, the amplitudes of the components are changed in the ratio σ, given by

$$\sigma_{n,i}=\frac{\bar{C}_{n,i}}{C_{n,i}}=\left[1-\gamma_i\left(1-\cos\frac{n\pi}{K}\right)\right]^{M_i}$$

if we consider a Fourier expansion for $2K$ equally spaced points, of the form

$$\psi_{i,j}=A_{o,i}+\sum_{n=1}^{K-1}C_{n,i}\cos\left(\frac{n\pi j}{K}+\alpha_{n,i}\right)+\frac{C_{K,i}}{2}\cos\left(\frac{\pi j}{K}\right).$$

Here $\alpha_{n,i}$ is the phase, $C_{n,i}$ and $\bar{C}_{n,i}$ are the amplitudes before and after smoothing and $A_{o,i}$ is the mean value of the field. Figure 3 shows σ as a function of wavelength at two high latitudes. Although a strong damping of the shorter wavelengths is evident at these latitudes, there is also considerable damping of the longer wavelengths.

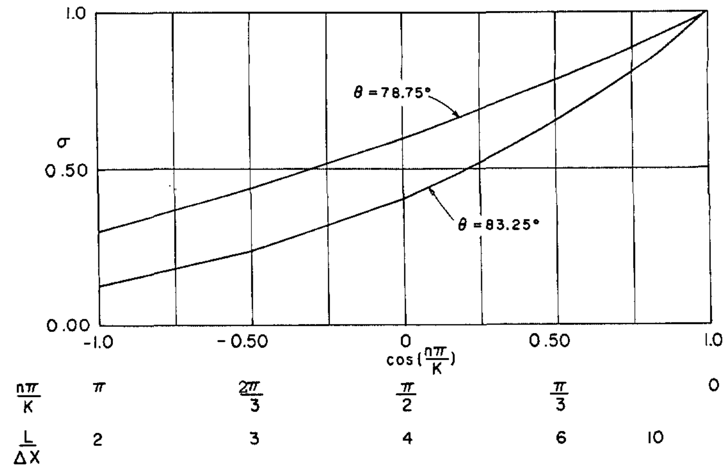

FIGURE 3.—Curves showing the variation of σ, the ratio of the smoothed to unsmoothed amplitudes, for $\theta=83.25°$ and $\theta=78.75°$ lat. L is the wavelength and Δx is the grid length in the east-west direction. $L/\Delta x$ thus represents the wavelengths in units of grid lengths.

In the second technique of smoothing, instead of performing weighted averaging, Fourier analyses of the same fields are made along latitude circles; this is followed by a reconstruction of the fields with the components of the shorter wavelengths truncated, that is,

$$C_{n,i}=0 \text{ for } n\geq\text{INT}\left(\frac{2K}{D_i}\right)$$

with an equivalent criterion for $2K+1$ points.

A reconstructed field is thus composed only of wavelengths greater than $\Delta\theta$ and these components are unaltered. Here also the mean value, the wave numbers, and the phases are not changed. The technique of Fourier filtering has also been used by Phillips (1959*a*) for the purpose of overcoming nonlinear computational instability.

4. NUMERICAL RESULTS AND CONCLUSIONS

For purposes of comparison in the analysis of the results, the resolution and integration scheme used in experiment 7 of the earlier study (Sankar-Rao and Umscheid 1969) were also adopted here in experiments 2 and 3. There are 20 points from pole to Equator (N=20) and the Kurihara grid is modified by adding an additional 8 points at each latitude ($KP=8$). A leapfrog time-integration scheme is used. The tests with smoothing in experiments 2 and 3 can then be compared with our earlier experiment 7 (reproduced here as experiment 1), for which no smoothing was used.

Table 2 gives the time step used in each experiment, a factor giving the relative computer time, and the range of total energy. Although energy conservation is not formally guaranteed when the smoothing schemes are employed, the global energy budget is still very accurate for the period of integration. It is important to note in this connec-

TABLE 2.—*The time step, the computer timing (normalized to 1 for experiment 1), and the range of the total energy* (RTE) *in percent of the initial value for the 10-day integration*

	Experiment	ΔT	Relative timing	RTE
		(s)		
No smoothing	1	60	1	4.2×10^{-8}
Weighted averaging	2	360	1/6	7.7×10^{-6}
Fourier filtering	3	600	1/5	4.4×10^{-6}

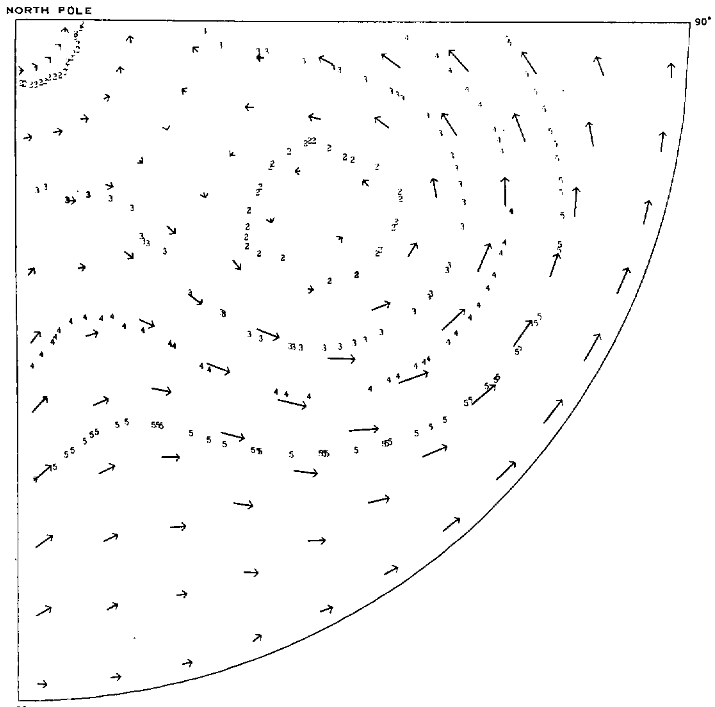

FIGURE 4.—Forecast height field and wind vectors at the end of 10 days for experiment 1; this is identical to experiment 7 of authors' previous paper (1969). The contour values decrease in steps of 0.5 km from contour 2=8.5 km.

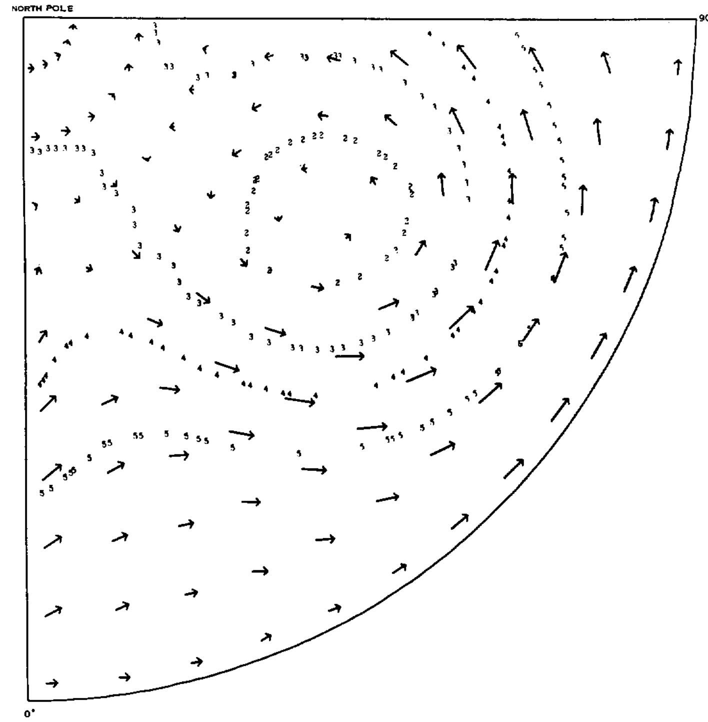

FIGURE 5.—Forecast height field and wind vectors at the end of 10 days for experiment 2; contour values are the same as in figure 4.

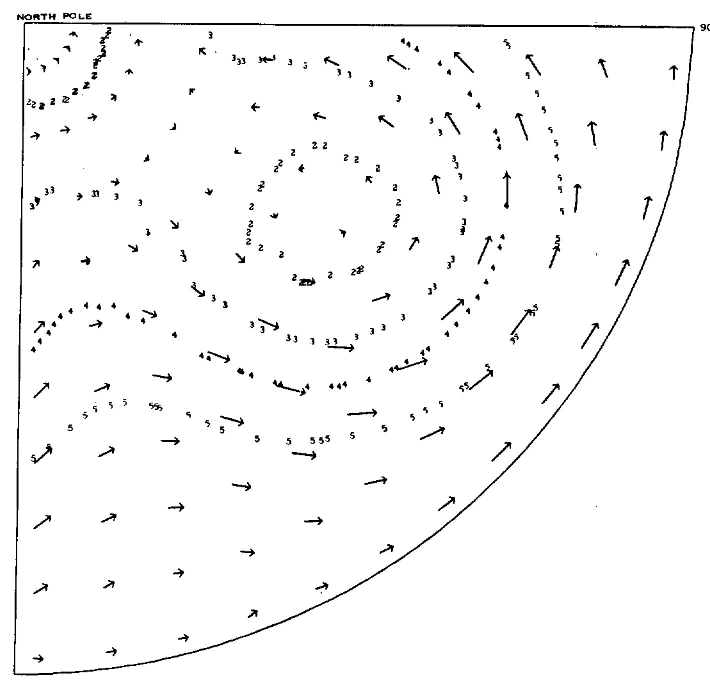

FIGURE 6.—Forecast height field and wind vectors at the end of 10 days for experiment 3; contour values are the same as in figure 4.

tion that the polar regions, where smoothing is performed, constitute less than 5 percent of the area of the globe.

Although experiment 3 permits the largest time step, a slightly better timing improvement is obtained for experiment 2 because of the lengthier computation required for the fast Fourier transforms used in the Fourier technique of experiment 3.

The geopotential and wind fields at the end of 10 days for the smoothing experiments are shown in figures 4, 5, and 6. It can be seen that the fields corresponding to experiments 1 and 3 are nearly identical. Experiment 2 is also very similar to experiment 1, although the fields are somewhat different in the higher latitudes. The similarities and differences in the mean zonal velocities, especially the appearance of easterlies in experiment 2, are displayed in figure 7.

As suggested by Dey (1969), the analytic steady-state solution corresponding to purely zonal motion is found to be accurately represented by the Kurihara grid and operators even without the addition of extra points and the accompanying smoothing. The results of experiment 4

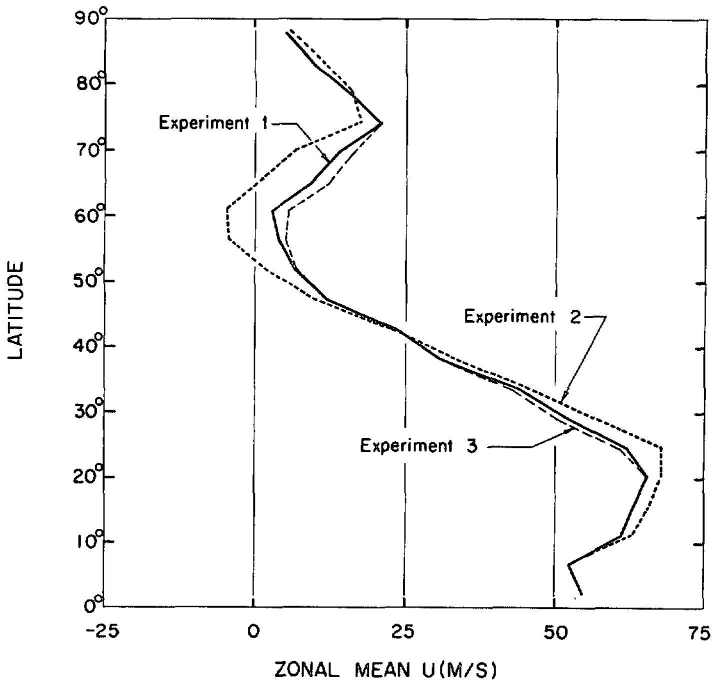

FIGURE 7.—Zonal mean east-west wind velocity (u) at the end of 10 days for experiments 1, 2, and 3. Westerly winds have positive speeds.

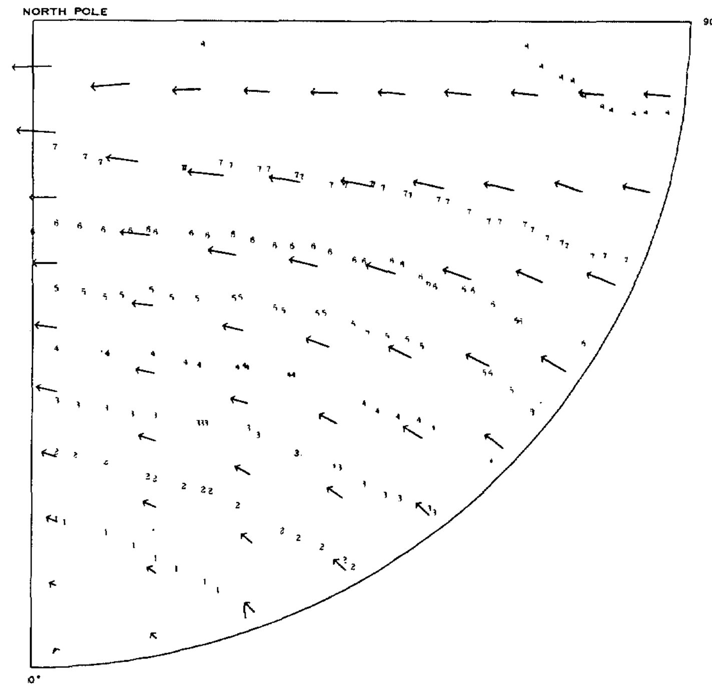

FIGURE 8.—Forecast height field and wind vectors at the end of 10 days for experiment 5; contour values increase in 30-m steps from contour 1=2.79 km.

indicate only very slight changes in the fields for the 10-day integration, and these fields are therefore not presented. Although changes do occur in the fields corresponding to cross-polar flow tested in experiment 5, the large-scale features of the flow are preserved for the 10-day integration period as can be seen from figure 8.

The results of this study indicate that the smoothing techniques tested may be a practical means of increasing computational efficiency in global numerical prediction without loss of accuracy in the large-scale features of the flow. In particular, the Kurihara operators and the modified Kurihara grid are rendered feasible for global integration of the complete primitive equations of atmospheric motion.

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